



# Free Body Dynamics of a Spinning Cylinder With Planar Restraint—(a.k.a. Barrel of Fun)

## Part II

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## Abstract

The dynamic motion of a cylinder is analyzed based on rotation about its center of mass and is restrained by a plane normal to the axis passing through its center of mass at an angle. The first part of this work presented an analysis of the stability of the motion. In the current report, the governing equations are numerically integrated in time and the steady state is obtained as a limit of the transient numerical solution. The calculated data are compared with observed behaviors.

## Introduction

The dynamic motion of rigid bodies is instructive, and sometimes, entertaining. Many books (e.g., Ref. 6) have been dedicated to space dynamics, rotordynamics, and other fields with vast industrial applications. However, many “funny” devices also present challenges to engineers. The spin reversal of the rattlebacks has been studied for a number of years [1–3]. The lifting of the axis of the spinning egg was modeled as well [4]. The first part of this paper [5], presented some analytical stability considerations regarding the motion of a cylinder with a planar restraint. The paper was inspired by the observation of the rolling and collapsing motions of an oil drum, operated by a skilled individual. A video clip of this phenomenon can be seen at <http://gltrs.grc.nasa.gov/cgi-bin/GLTRS/browse.pl?2006/TM-2006-213583/drumroll1.avi>. The motion can be induced into the drum by a regular periodic change in torque on the periphery of the drum, such as once every time the drum rotates about its transverse axis (see the video clip). The control of the motion definitely requires much skill in rhythm and agility. Even though difficult to describe and put into practice, such motions are of interest to rotordynamicists in stabilizing and maintaining stable spinning spacecraft and in spontaneous unloading of transmissions.

The outcome compares favorably with known visual data. However, we recognize several limitations, as maintenance torques, friction, and other dissipation mechanisms are not completely addressed. The current analysis is not dedicated to the transient period shown in the video clip. The first part of this paper [5] was concerned with the stability of the barrel after a stationary situation has been reached. Here we integrate numerically the equations of motion and obtain the steady state as a limit of the transient solution. Some issues related to the control and to the startup of the motion are yet to be modeled. Future papers will address different aspects which are not treated herein.

## Nomenclature

$\psi$	$\equiv$	general coordinate angle
$\psi_t$	$\equiv$	$d\psi/dt$
$\psi_{tt}$	$\equiv$	$d^2\psi/dt^2$
$\psi_j$	$\equiv$	$\psi$ along $j$ -coordinate axis
$t$	$=$	time

Figures 1 to 3 show the geometry, angles, and rotations of the frames of the axes.

The position of the mass center is  $L^* = \xi L/2$  where  $\xi$  is the location of the center of mass measured from the base upward.

$\chi$	$=$	$(L^* \sin \alpha - R \cos \alpha)$
$R$	$=$	radius
$L$	$=$	length
$\alpha$	$=$	angle of inclination or nutation angle
$v$	$=$	precession angle of the geometrical elements
$\varphi$	$=$	spin angle

# Analysis

## Preliminary Aspects

The equations of motion of the cylinder can be obtained in various ways. In this study they were obtained by applying the angular momentum equation. The complete derivation of the equation of motion is described in Ref. 5. However, the main elements will be summarized below, for a better readability of this report. Note that, with fixed ends or heads, the cylinder is termed a “drum”; without a closed upper end, an “open-drum”; and without either end closed, is simply referred to as a “plain cylinder.” In all cases the restraint is that of a flat surface upon which the cylinder is supported and rotates. In order to support the motion, the cylinder must spin on multiple axes.

The current report employs the following assumptions [5]:

- (1) The drum is modeled as a symmetric rigid body whose center of mass is fixed during the motion and restrained from below by a plane surface.
- (2) The center of mass is located on the centerline of the barrel.
- (3) The motion is described by the moment of momentum or angular momentum theorem. The equations will be written in the body-fixed frame  $OXYZ$  (see Fig. 1).
- (4) The frame  $OX_FY_FZ_F$  is fixed in space.
- (5) The frame  $OXYZ$  is fixed with respect to the drum and executes all the motions of that drum.

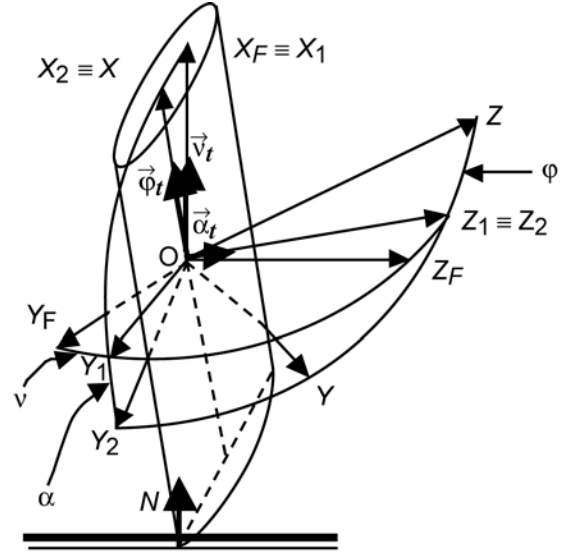
The order of rotations necessary to obtain the  $OXYZ$  frame from  $OX_FY_FZ_F$  (Fig. 1) is schematically written as

$$OX_FY_FZ_F \xrightarrow[\frac{v}{OX_F}]{\frac{v}{OX_F}} OX_1Y_1Z_1 \xrightarrow[\frac{\alpha}{OZ_1}]{\frac{\alpha}{OZ_1}} OX_2Y_2Z_2 \xrightarrow[\frac{\phi}{OX_2}]{\frac{\phi}{OX_2}} OXYZ$$

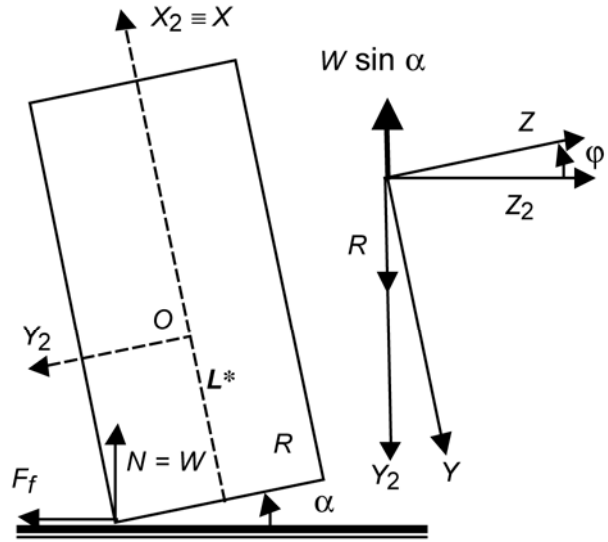
Two types of friction act upon the drum. The rolling friction is perpendicular to the  $OY_2$  axes and opposes the precession of the drum. The sliding friction is parallel to  $OY_2$  and opposes the increase of the nutation angle,  $\alpha$  (Fig. 2). The rolling friction is much smaller than the sliding friction, so in the subsequent analyses is neglected (in industrial gyroscopic applications this is usually compensated by an electric motor). The components of the forces  $F$  and  $N$ , where  $F_f$  is the friction force and  $W$  the weight, are (Fig. 2)

$$\begin{aligned} [F_{fX} \ F_{fY} \ F_{fZ}]^T &= F_f [\sin \alpha, \cos \alpha \cos \phi, -\cos \alpha \sin \phi]^T \\ [N_X \ N_Y \ N_Z]^T &= W [\cos \alpha, -\sin \alpha \cos \phi, \sin \alpha \sin \phi]^T \end{aligned} \quad (1)$$

and the components of the angular speed along the body-fixed axes  $OXYZ$  are (Fig. 3)



**Figure 1.—Position angles ( $\alpha$ ,  $\phi$ , and  $\nu$ ) and rotations of frames of axes ( $X$ ,  $Y$ , and  $Z$ ) under normal load  $N$  for spinning cylinder.**



**Figure 2.—Force components ( $N$ ,  $F$ ,  $W$ ), position vectors ( $\vec{X}$ ,  $\vec{Y}$ ,  $\vec{Z}$ ), position angles ( $\phi$ ,  $\alpha$ ), and geometric parameters ( $R$ ,  $L^*$ ) for spinning cylinder.**

$$\begin{aligned} \omega_X &= \phi_t + \nu_t \cos \alpha \\ \omega_Y &= \alpha_t \sin \phi - \nu_t \cos \alpha \cos \phi \\ \omega_Z &= \alpha_t \cos \phi + \nu_t \sin \alpha \sin \phi \end{aligned} \quad (2)$$

The components of the position vector  $[\vec{r}]$  of the acting point of the forces are (Figs. 1 and 2)

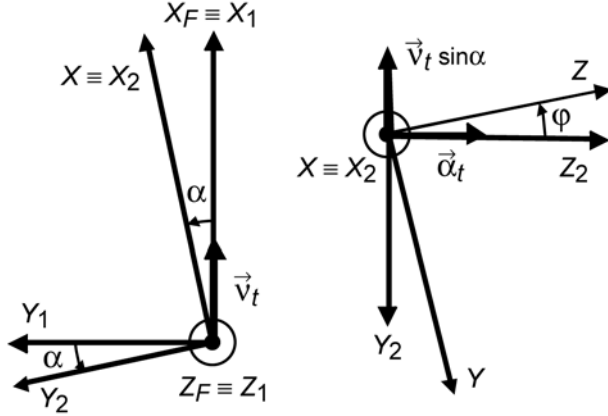


Figure 3.—Components of angular speed for spinning cylinder.

$$[r_x \quad r_y \quad r_z]^T = [-L^* \quad R \cos \phi \quad -R \sin \phi]^T \quad (3)$$

## Governing Equations

The moment of momentum equations around the fixed mass center are written in terms of the body axes  $OXYZ$ :

$$\left\{ \frac{d\vec{K}_0}{dt} \right\}_{XYZ} \equiv \left\{ \frac{\partial \vec{K}_0}{\partial t} + \vec{\omega} \times \vec{K}_0 \right\}_{XYZ} = \{\vec{M}\}_{XYZ} \quad (4)$$

The moments produced by the normal reaction and by the friction are

$$\begin{aligned} M_X &= 0 \\ M_Y &= W \sin \phi (L^* \sin \alpha - R \cos \alpha) \\ &\quad - F_f \sin \phi (L^* \cos \alpha + R \sin \alpha) \\ M_Z &= W \cos \phi (L^* \sin \alpha - R \cos \alpha) \\ &\quad + F_f \cos \phi (L^* \cos \alpha + R \sin \alpha) \end{aligned} \quad (5)$$

Next,

$$\vec{K}_0 = [J_X \omega_X \quad J \omega_Y \quad J \omega_Z]^T \quad (6)$$

and

$$\{\vec{\omega} \times \vec{K}_0\} = \begin{Bmatrix} 0 \\ J_X \omega_X \omega_Z - J \omega_X \omega_Z \\ J \omega_X \omega_Y - J_X \omega_X \omega_Y \end{Bmatrix} \quad (7)$$

where  $J$  is the moment of inertia. Collecting the terms, the equations of motion of the drum are obtained:

$$\begin{aligned} \frac{d\omega_X}{dt} &= \frac{M_X}{J_X} \\ \frac{d\omega_Y}{dt} &= \frac{M_Y}{J} + \frac{J - J_X}{J} \omega_X \omega_Z \\ \frac{d\omega_Z}{dt} &= \frac{M_Z}{J} + \frac{J_X - J}{J} \omega_X \omega_Y \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{dv}{dt} &= \frac{1}{\sin \alpha} (\omega_Z \sin \phi - \omega_Y \cos \phi) \\ \frac{d\alpha}{dt} &= \omega_Y \sin \phi + \omega_Z \cos \phi \\ \frac{d\phi}{dt} &= \omega_X - \frac{1}{\tan \alpha} (\omega_Z \sin \phi - \omega_Y \cos \phi) \end{aligned} \quad (9)$$

where the “kinematic equations”, (i.e., Eq. (9)), can be readily obtained by solving Eq. (2) for  $\phi$ ,  $\alpha$ , and  $v$ .

An analysis developed in Ref. 5 showed that, in an ideal, frictionless case, the motion is stable provided that

$$\omega_X > \frac{2\sqrt{WL^*J}}{J_X} \quad (10)$$

It was also indicated that the above equation is also a good stability warning for more realistic cases, when the sliding friction is present.

In the first part of this paper [5], the stability conditions for barrels with and without lids and with various length/diameter ( $L/D$ ) ratios were computed. The results are summarized in Table 1 using moments of inertia from Table 2.

TABLE 1.—THEORETICAL STABILITY LIMITS FOR SPINNING BARRELS

Geometry	Length/diameter ratio, $L/D^a$					
	1.0		1.5		2.0	
	Angular speed, <sup>b</sup> $\omega_x$ , rad/s	Weight, $W$ , N	Angular speed, <sup>b</sup> $\omega_x$ , rad/s	Weight, $W$ , N	Angular speed, <sup>b</sup> $\omega_x$ , rad/s	Weight, $W$ , N
Drum	13.8	122.5	20.3	163.3	27.8	204.1
Open drum	10.8	102.0	16.7	142.9	23.5	183.7
Plain cylinder	10.6	81.6	15.9	122.5	22.3	163.3

<sup>a</sup> $D = 0.58$  M.

<sup>b</sup> $\omega_x$  = minimum stable speed.

TABLE 2.—MOMENT OF INERTIA DATA  
FOR BARRELS WITH LENGTH 0.87 m  
AND DIAMETER 0.58 m.

Geometry	Moment of inertia		Weight, W, N
	$J_x$ , kg·m <sup>2</sup>	$J_z$ , kg·m <sup>2</sup>	
Drum	1.225	2.187	163.3
Open drum	1.138	1.694	142.9
Cylinder	1.050	1.313	122.5

The next the next section presents some investigations of the barrel motion performed via the numerical integration of the equations of motion (Eqs. (8) and (9)).

## Results and Discussion

In general, bodies with a gyroscopic motion become unstable at high nutation angle, so the small angle stability condition is usually a good indicator for the stability domain. However, the solution of the equations of motion can generate further insight, which is inherently eliminated in the approximate analytical studies.

As it is well known, the equations of motion for a rigid body can be solved analytically only in a few cases, so the studies can only be done numerically. In this section we will present some numerical solutions of the current model for the drum, for various conditions, above and below the theoretical stability limit Eq. (10), deduced in Ref. 5. The integration is performed using a fourth-order Runge-Kutta algorithm. The variable  $\omega_x$ , which is basically imposed by the operator of the drum (see the following Video Clip section), was used as a control variable and it was gradually increased until it reached the desired steady state value,  $\omega_X = \omega_X^{\text{STEADY}}(1 - \exp(-bt))$ , where  $b$  is an arbitrary constant. The remaining equations were integrated numerically, using a regular fourth-order Runge Kutta method; that is, for an arbitrary system

$$\dot{y}_i = f_i(t, y_1, \dots, y_n) \quad i = 1, \dots, n \quad (11)$$

the solution is calculated using the usual

$$y_i(t + \Delta t) = y_i(t) + \frac{1}{6}(k_{1i} + k_{2i} + k_{3i} + k_{4i}) \quad i = 1, \dots, n \quad (12)$$

with

$$\begin{aligned} k_{1i} &= \Delta t \cdot f_i(t, y_1, \dots, y_n) \\ k_{2i} &= \Delta t \cdot f_i\left(t, y_1 + \frac{k_{11}}{2}, \dots, y_n + \frac{k_{1n}}{2}\right) \\ k_{3i} &= \Delta t \cdot f_i\left(t, y_1 + \frac{k_{21}}{2}, \dots, y_n + \frac{k_{2n}}{2}\right) \\ k_{4i} &= \Delta t \cdot f_i\left(t, y_1 + k_{31}, \dots, y_n + k_{3n}\right) \end{aligned} \quad i = 1, \dots, n \quad (13)$$

Three types of cases were investigated. Figures 4(a) to (f) illustrate cases when the steady-state  $\omega_X$  is half of the theoretical stability value (Eq. (10)). The results presented in Figs. 5(a) to (d) were obtained for a steady angular speed equal to the value indicated by Eq. (10), and Figs. 6(a) to (f) present cases when the steady  $\omega_X$  is 3 and 4 times larger than the theoretical stability limit.

Figures 4(a) and (b) present results obtained by the integration of the general Eqs. (8) and (9) for a case when the friction was considered zero and the variable  $\omega_X$  was gradually increased until it reached a steady-state value equal to half the critical value determined by Eq. (10). The nutation angle oscillates steadily (Fig. 4(a)), and the orbit of the center point of the top lid (Fig. 4(b)), presents a dominant of the high-amplitude nutation, which is not really visible in the real motion.

Figures 4(c) to (f) are obtained for the same dynamic conditions as Figs. 4(a) and (b), but here a coulombian dry friction was taken into account. The coefficient of friction between the drum and the pavement was 0.2. The oscillation of the nutation angle (Fig. 4(c)) does not contain a clear dominant frequency as in the previous case and the precession, (Fig. 4(d)) increases almost linearly. Figure 4(e) shows the variation of the control variable,  $\omega_X = \omega_X^{\text{STEADY}}(1 - \exp(-bt))$ . The steady-state value varies from case to case, but the shape of the curve is the same for all the results presented herein. Figure 4(f) shows the top view of the orbit of the center of the top lid of the barrel. The orbits present large circular patterns which are qualitatively comparable to the motion of the drum before the steady state is reached.

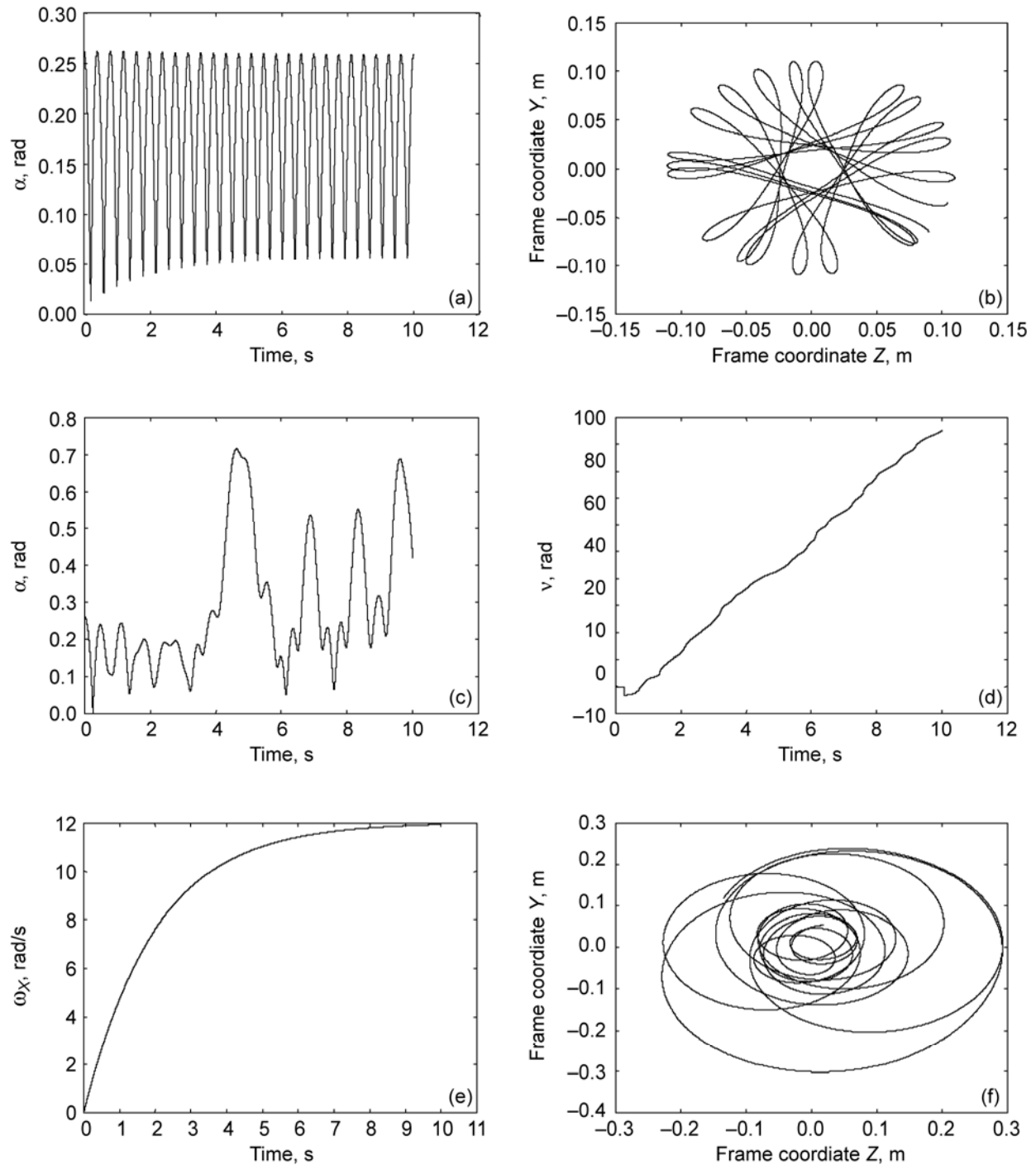
The same types of behaviors encountered in Figs. 4(a) to (f) are illustrated in Figs. 5(a) to (d). The first two figures present the case when friction was neglected. The last two images (Figs. 5(c) and (d)), show the case when the friction was present. As before, the last two figures can be considered more realistic for the motion of the drum before the steady state is reached.

Figures 6 and 7 present results obtained for high-speed cases. The nutation angle oscillates steadily around a median position, as shown in Figs. 6(a), (c), and 7(a). The orbits of the center point of the top lids are shown in Figs. 6(b), (d), and 7(b). This orbit series shows fast, small oscillations that are “wrapped” around a circle. For the case of fast motion, the influence of friction was found to be less important than before the steady state was reached.

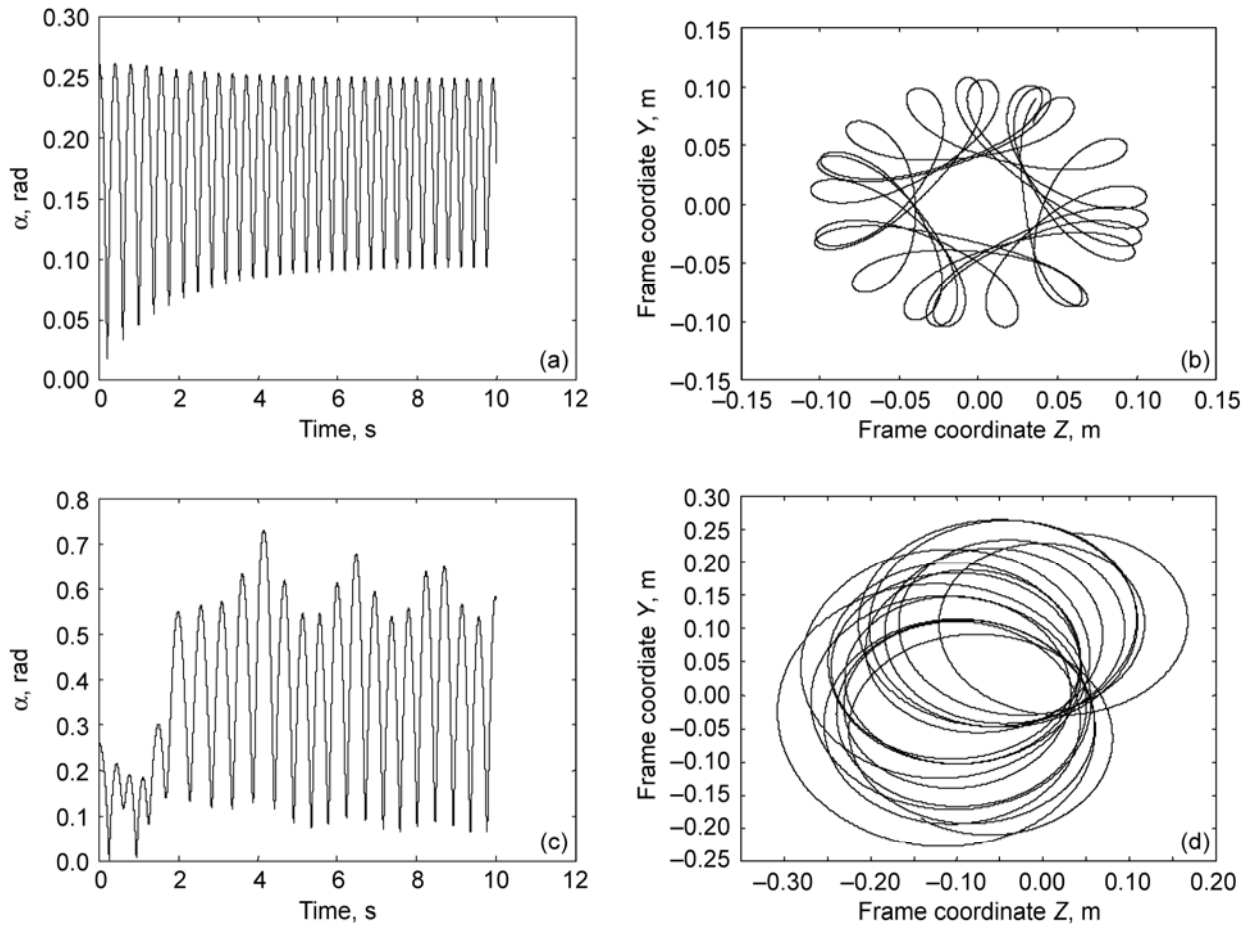
Before concluding, it is noted that, as expected, the numerical results indicate that friction has an important role, especially in the initial stage of the motion. This result was also confirmed (e.g., Ref. 4).

## Video Clip

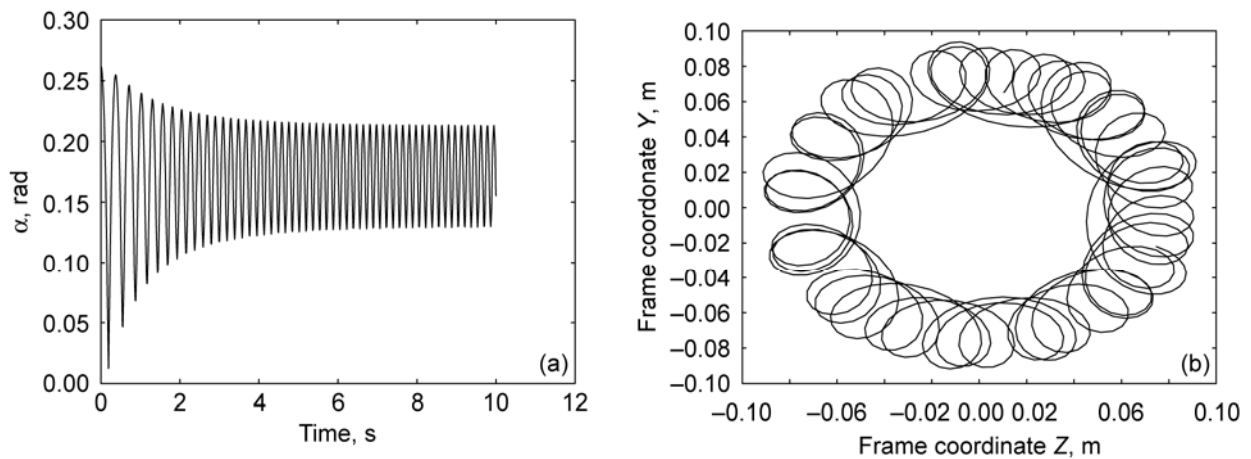
There is a video (from Figure 10 in the report NASA/TM—2006-213583, which can be accessed on the Web at <http://www.sti.nasa.gov>) that illustrates the skill of a street merchant walking his empty open-end 55-gallon drum about a heavily trafficked street as one might walk a dog, yet with near perfect control. The drum motion and feedback control is



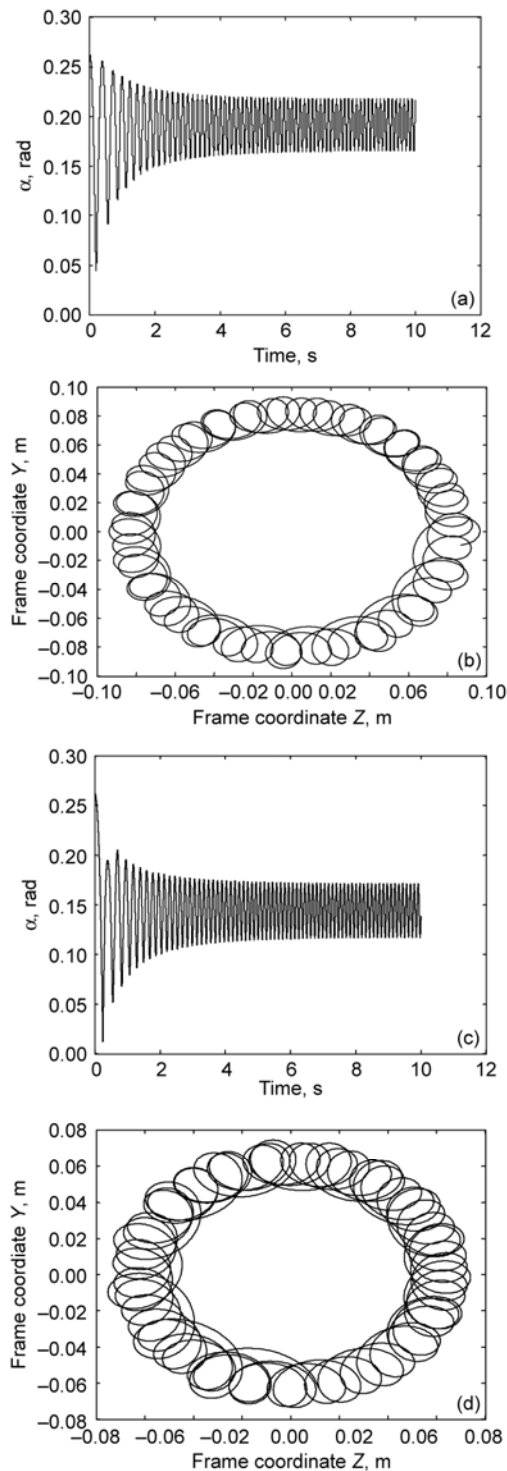
**Figure 4.—Motion of rotating drum when steady-state angular speed  $\omega_X = 0.5 (2\sqrt{JWL^*})/J_X$ . (a) Variation of nutation angle  $\alpha$ ; no friction. (b) Orbit of center of top lid of drum, top view; no friction. (c) Variation of nutation angle  $\alpha$ ; friction coefficient is 0.2. (d) Variation of precession angle  $\nu$ ; friction coefficient is 0.2. (e) Variation of angular speed  $\omega_X$ ; qualitatively valid for all cases (plain cylinder, open drum, and drum). (f) Orbit of center of top lid of drum, top view; friction coefficient is 0.2.**



**Figure 5.—Motion of rotating drum when steady-state angular speed  $\omega_X = (2\sqrt{JWL^*})/J_X$ . (a) Variation of nutation angle  $\alpha$ ; no friction. (b) Orbit of center of top lid of drum, top view; no friction. (c) Variation of nutation angle  $\alpha$ ; friction coefficient is 0.2. (d) Orbit of center of top lid of drum, top view; friction coefficient is 0.2.**



**Figure 6.—Motion of rotating drum when steady-state angular speed  $\omega_X = 3(2\sqrt{JWL^*})/J_X$ . (a) Variation of nutation angle  $\alpha$ ; no friction. (b) Orbit of center of top lid of drum, top view; no friction.**



**Figure 7.—Motion of rotating drum when steady-state angular speed  $\omega_X = 4 (2\sqrt{JWL^*})/J_X$ . (a) Variation of nutation angle  $\alpha$ ; no friction. (b) Orbit of center of top lid of drum, top view; no friction. (c) Variation of nutation angle  $\alpha$ ; friction coefficient is 0.2. (d) Orbit of center of top lid of drum, top view; friction coefficient is 0.2.**



**Figure 8.—Lifting the barrel (motion of barrel during transient stage). Taken from video clip.**

maintained by a cadence of regular hand imparted surface torques at each rotation of the barrel. Small angles from vertical require short slaps and large angles require complex upward motions of the hand and body. Figure 8 presents some still-shots taken in the transient period during the lifting of the barrel.

## Conclusions

The dynamic motion of a drum about its center of mass that is restrained by a plane normal to the axis of rotation passing through its center of mass at an angle  $\alpha$  is both entertaining and instructive. The first part of this paper presented some analytical considerations regarding the stability of such a barrel. Here, some aspects of the motion using the numerical solution of the equations of motion were discussed. The numerical results, obtained both above and below the theoretical stability limit, confirm the validity of the analytical results obtained in the first part of this work. The numerical results also indicate that friction has an important role, especially in the initial stage of the motion.

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